

The radiative potential method for calculations of radiative corrections to phenomena of many-electron atoms; parity nonconservation in cesium

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Parity nonconservation in cesium

- Parity nonconservation (PNC) measured in $6s$ - $7s$ E1 transition in cesium with 0.35% precision [Wood et al. ('97)]
- Sensitive to nuclear weak charge Q_W

Standard Model: $Q_W = -N + Z(1 - 4 \sin^2 \theta_W)$ (tree)

$$\approx -N$$

$$Q_W^{SM}({}_{55}^{133}\text{Cs}) = -73.19(13) \quad [\text{Marciano, Rosner}]$$

- Q_W extracted from measurements using atomic structure calculations

$$\begin{aligned} E_{PNC} &= \sum_n \left[\frac{\langle 7s | D | np \rangle \langle np | H_W | 6s \rangle}{E_{6s} - E_{np}} + \frac{\langle 7s | H_W | np \rangle \langle np | D | 6s \rangle}{E_{7s} - E_{np}} \right] \\ &= \xi Q_W \end{aligned}$$

Recent developments in atomic theory:

Atomic theory	Q_W	$Q_W - Q_W^{SM}$	Ref.
1%-precision MBPT calcs. of Dzuba et al. (1989), Blundell et al. (1990,1992)	$-72.11(27)_{\text{exp}}(89)_{\text{theor}}$	1.2σ	Wood et al. (1997)
Re-interpretation of atomic theory error, 1% \rightarrow 0.4%	$-72.06(26)_{\text{exp}}(34)_{\text{theor}}$	2.5σ	Bennett, Wieman (1999)
+ Breit interaction	+0.6%		Derevianko (2000), Dzuba et al. (2001), Kozlov et al. (2001)
+ Strong-field vacuum polarization	-0.4%		Johnson et al. (2001), Milstein, Sushkov (2002)
+ Neutron skin	+0.2%		Derevianko (2002)
0.5%-precision MBPT calc., incl. all above corrections	$-72.16(29)_{\text{exp}}(36)_{\text{theor}}$	2.1σ	Dzuba et al. (2002)
+ Strong-field QED self-energy and vertex corrections (weak matrix elements)	+0.8%		Kuchiev, Flambaum (2002), Milstein et al. (2002, 2003), Sapirstein et al. (2003)
TOTAL	$-72.74(29)_{\text{exp}}(36)_{\text{theor}}$	0.9σ	

Not final story...

Need calculation of strong-field radiative corrections to full PNC amplitude (not only weak matrix elements)

- Recent calculation in effective atomic potential,

$$\delta E_{PNC} = (0.41 - 0.67(3))\% = -0.27(3)\% \quad [\text{Shabaev et al. ('05)}]$$

However, many-body corrections could be large!

Our aim: *calculate radiative corrections to PNC amplitude with full account of many-body effects*

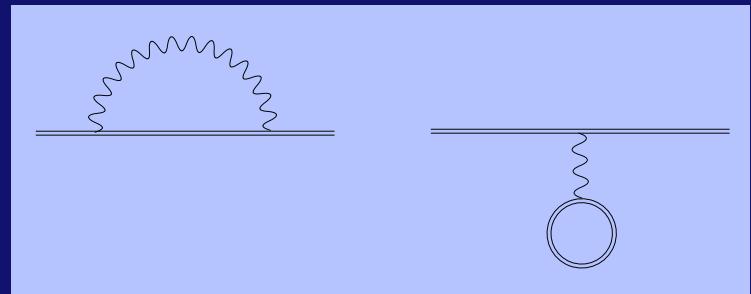
- 98% of PNC amplitude from n=6,7,8,9 :
calculate radiative corrections to energies and EI amplitudes;
take corrections to weak matrix elements from previous works

Radiative potential

- Construct radiative potential that can be added to atomic potential in many-body procedures

Radiative potential L defined s.t.

$$\delta E_n = \langle n | L(r, r', E) | n \rangle$$



Shifts in neutral atoms:

- Lamb shift arises due to interactions at small distances where electrons are unscreened
- Energies of valence electrons in neutral atoms $E \sim 10^{-5} mc^2$
⇒ atomic electron wave functions are proportional to Coulomb wave functions with high n

$$\langle \psi | L | \psi \rangle_{\text{neutral}} = \langle \psi | L | \psi \rangle_{\text{H-like}} \frac{\rho(0)_{\text{neutral}}}{\rho(0)_{\text{H-like}}}$$

Use approximate, local radiative potential

$$\Phi_{\text{rad}}(r) = \Phi_U(r) + \frac{2}{3}\Phi_{WC}^{\text{simple}}(r) + \Phi_g(r) + \Phi_f(r) + \Phi_l(r)$$

- *Ab initio* derivation for $Z\alpha \ll 1$
- Refined to include higher orders in $Z\alpha$ by fitting to (Mohr *et al.*) Lamb shifts for high states in H-like ions for $10 \leq Z \leq 110$

e.g.

$$\Phi_f(r) = -A(Z, r) \frac{\alpha}{\pi} \Phi(r) \int_1^\infty dt \frac{1}{\sqrt{t^2 - 1}} \left[\left(1 - \frac{1}{2t^2}\right) \left(\ln(t^2 - 1) + 4 \ln(1/Z\alpha + 0.5) \right) - \frac{3}{2} + \frac{1}{t^2} \right] e^{-2trm}$$

$$\Phi_l(r) = -\frac{B(Z)}{e} Z^4 \alpha^5 m c^2 e^{-Zr/a_B}$$

%-difference between our self-energy results and those of Mohr, Kim for H-like ions

Z	10	20	30	40	50	60	70	80	90	100	110
$5s_{1/2}$	0.0	0.4	0.5	0.3	0.0	-0.2	-0.2	0.0	0.1	0.1	0.0
$5p_{1/2}$	-0.8	-3.6	-2.8	-1.8	-1.1	-0.7	-0.3	0.1	0.8	1.8	3.3
$5p_{3/2}$	-2.5	-8.3	-8.9	-7.3	-5.2	-3.1	-1.1	0.4	1.4	1.7	0.8

Energies for cesium

Add radiative potential to atomic procedures, $\Phi_{DHF} + \Phi_{\text{rad}}$

Radiative corrections to ionization energies (cm⁻¹)

	6s _{1/2}	7s _{1/2}	6p _{1/2}	7p _{1/2}	8p _{1/2}	9p _{1/2}
(DHF) ₀	15.5	4.3	0.2	0.07	0.03	0.02
(DHF) ₀ + δV	15.9	4.3	-0.8	-0.3	-0.1	-0.07
(DHF) ₀ + δV + $\hat{\Sigma}^{corr}$	17.6	4.1	-0.4	-0.1	-0.05	-0.03

- $(DHF)_0$ - Dirac-Hartree-Fock field without relaxation
 δV - core relaxation
 Σ^{corr} - correlations

c.f. *ab initio* QED calculations for cesium 6s in effective atomic potentials:

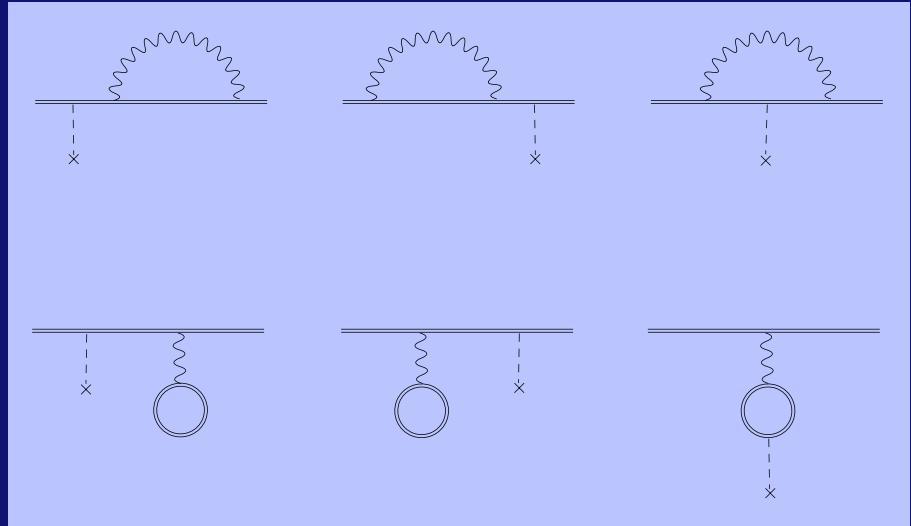
15-27 cm⁻¹ [Labzowsky et al., '99]

13-23 cm⁻¹ [Sapirstein and Cheng, '02]

El amplitudes

Low-energy theorem:

$$\begin{aligned} & \langle 1|D|2\rangle_{\text{rad}} \\ &= \sum_n \frac{\langle 1|D|n\rangle \langle n|\Sigma(E_2)|2\rangle}{E_2 - E_n} + \sum_n \frac{\langle 1|\Sigma(E_1)|n\rangle \langle n|D|2\rangle}{E_1 - E_n} \\ & - \frac{1}{2} \langle 1|D \frac{\partial \Sigma}{\partial E} + \frac{\partial \Sigma}{\partial E} D|2\rangle + \frac{1}{2} \langle 1|D|2\rangle (\langle 1| \frac{\partial \Sigma}{\partial E} |1\rangle + \langle 2| \frac{\partial \Sigma}{\partial E} |2\rangle) \end{aligned}$$



Vertex and normalization contributions are small, $\frac{\partial \Sigma}{\partial E} \sim \frac{\Sigma}{\omega} \sim \frac{1}{Z^2} \frac{\Sigma}{E}$

Relative contributions from radiative potential: $\alpha^3 Z^2 \ln(1/\alpha^2 Z^2)$

Other contributions: $\alpha^3 (Z_i + 1)^2$, Z_i - ion charge

In neutral atoms radiative potential contribution is $\sim Z^2 \times$ larger!

E1 amplitudes for cesium

Relative radiative corrections R_{sp} to E1 amplitudes, $\langle s|r|p \rangle = \langle s|r|p \rangle_0 (1 + \alpha/\pi R_{sp})$

	$6s-6p$	$6s-7p$	$6s-8p$	$6s-9p$	$7s-6p$	$7s-7p$	$7s-8p$	$7s-9p$
DHF	0.266	-2.90	-4.62	-5.68	-0.451	0.270	-2.07	-3.21
DHF+RPAE	0.286	-4.39	-11.9	-29.7	-0.432	0.270	-2.20	-3.60
DHF+RPAE+ $\hat{\Sigma}^{corr}$	0.265	-2.91	-6.25	-10.3	-0.340	0.231	-1.60	-2.52

DHF - Dirac-Hartree-Fock (with relaxation)

RPAE - random phase approximation with exchange

c.f. *ab initio* QED calculation for Cs 6s-6p amplitude in effective atomic potential

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TABLE II. Self-energy contributions R_{wv} to E1 matrix elements for $ns_{1/2}-np_{1/2}$ transitions in the alkali metals: error of 0.001 for sum. Units, $(\alpha/\pi)z_{wv}$.

Term	Li	Na	K	Rb	Cs	Fr
$R_{wv}(V;00)$	-9.162	-9.308	-9.509	-9.573	-9.655	-9.647
$R_{wv}(V;iw)$	134.712	105.062	127.516	125.136	132.719	112.464
$R_{wv}(V;\text{poles})$	-137.256	-107.481	-129.741	-127.273	-134.877	-114.624
$R_{wv}(D)$	11.698	11.712	11.731	11.738	11.748	11.747
$R_{wv}(\text{PO};ns)$	0.003	0.031	0.067	0.182	0.326	0.787
$R_{wv}(\text{PO};np)$	0.000	0.001	0.000	0.000	0.000	0.202
Sum	-0.005	0.017	0.064	0.210	0.261	0.929

Radiative corrections to PNC in cesium

$$\begin{aligned}\delta E_{PNC} &= -0.34\% \text{ (energies)} + 0.43\% \text{ (E1)} - 0.41\% \text{ (weak)} \\ &= (-0.32 \pm 0.03)\%\end{aligned}$$

c.f. $(-0.27 \pm 0.03)\%$ [Shabaev *et al.* ('05)]

- Correcting atomic structure calculation [Dzuba, Flambaum, Ginges ('02)]

$$\Rightarrow E_{PNC} = -0.898(1 \pm 0.5\%) \times 10^{-11} i e a_B (-Q_W/N)$$

$$\begin{aligned}\Rightarrow \text{nuclear weak charge } Q_W &= -72.66(29)_{\text{exp}}(36)_{\text{theor}} \\ \text{agrees with standard model } Q_W - Q_W^{\text{SM}} &= 0.53(48)\end{aligned}$$

End of story!